

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

## MATHEMATICS

9709/07
Paper 7 Probability \& Statistics 2 (S2)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet. Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 The random variable $X$ has the distribution $\mathrm{B}(10,0.15)$. Find the probability that the mean of a random sample of 50 observations of $X$ is greater than 1.4.

2 The random variable $X$ has the distribution $\mathrm{N}\left(3.2,1.2^{2}\right)$. The sum of 60 independent observations of $X$ is denoted by $S$. Find $\mathrm{P}(S>200)$.

3 A machine has produced nails over a long period of time, where the length in millimetres was distributed as $\mathrm{N}(22.0,0.19)$. It is believed that recently the mean length has changed. To test this belief a random sample of 8 nails is taken and the mean length is found to be 21.7 mm . Carry out a hypothesis test at the $5 \%$ significance level to test whether the population mean has changed, assuming that the variance remains the same.

4 At a certain airport $20 \%$ of people take longer than an hour to check in. A new computer system is installed, and it is claimed that this will reduce the time to check in. It is decided to accept the claim if, from a random sample of 22 people, the number taking longer than an hour to check in is either 0 or 1 .
(i) Calculate the significance level of the test.
(ii) State the probability that a Type I error occurs.
(iii) Calculate the probability that a Type II error occurs if the probability that a person takes longer than an hour to check in is now 0.09 .

5 It is proposed to model the number of people per hour calling a car breakdown service between the times 0900 and 2100 by a Poisson distribution.
(i) Explain why a Poisson distribution may be appropriate for this situation.

People call the car breakdown service at an average rate of 20 per hour, and a Poisson distribution may be assumed to be a suitable model.
(ii) Find the probability that exactly 8 people call in any half hour.
(iii) By using a suitable approximation, find the probability that exactly 250 people call in the 12 hours between 0900 and 2100.

6 The daily takings, $\$ x$, for a shop were noted on 30 randomly chosen days. The takings are summarised by $\Sigma x=31500, \Sigma x^{2}=33141816$.
(i) Calculate unbiased estimates of the population mean and variance of the shop's daily takings.
(ii) Calculate a $98 \%$ confidence interval for the mean daily takings.

The mean daily takings for a random sample of $n$ days is found.
(iii) Estimate the value of $n$ for which it is approximately $95 \%$ certain that the sample mean does not differ from the population mean by more than $\$ 6$.
$7 \quad$ The continuous random variable $X$ has probability density function given by

$$
\mathrm{f}(x)= \begin{cases}\frac{3}{4}\left(x^{2}-1\right) & 1 \leqslant x \leqslant 2 \\ 0 & \text { otherwise }\end{cases}
$$

(i) Sketch the probability density function of $X$.
(ii) Show that the mean, $\mu$, of $X$ is 1.6875 .
(iii) Show that the standard deviation, $\sigma$, of $X$ is 0.2288 , correct to 4 decimal places.
(iv) Find $\mathrm{P}(1 \leqslant X \leqslant \mu+\sigma)$.

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